Using reduced-order models to study dynamic legged locomotion:

Parameter identification and model validation

Sam Burden, Shai Revzen, Talia Moore,
Shankar Sastry, and Robert Full



Generating testable hypotheses is challenging







- Mechanical system has many degrees-of-freedom
- Motion imposes changing reference frames
- Perturbations induce nonlinear effects

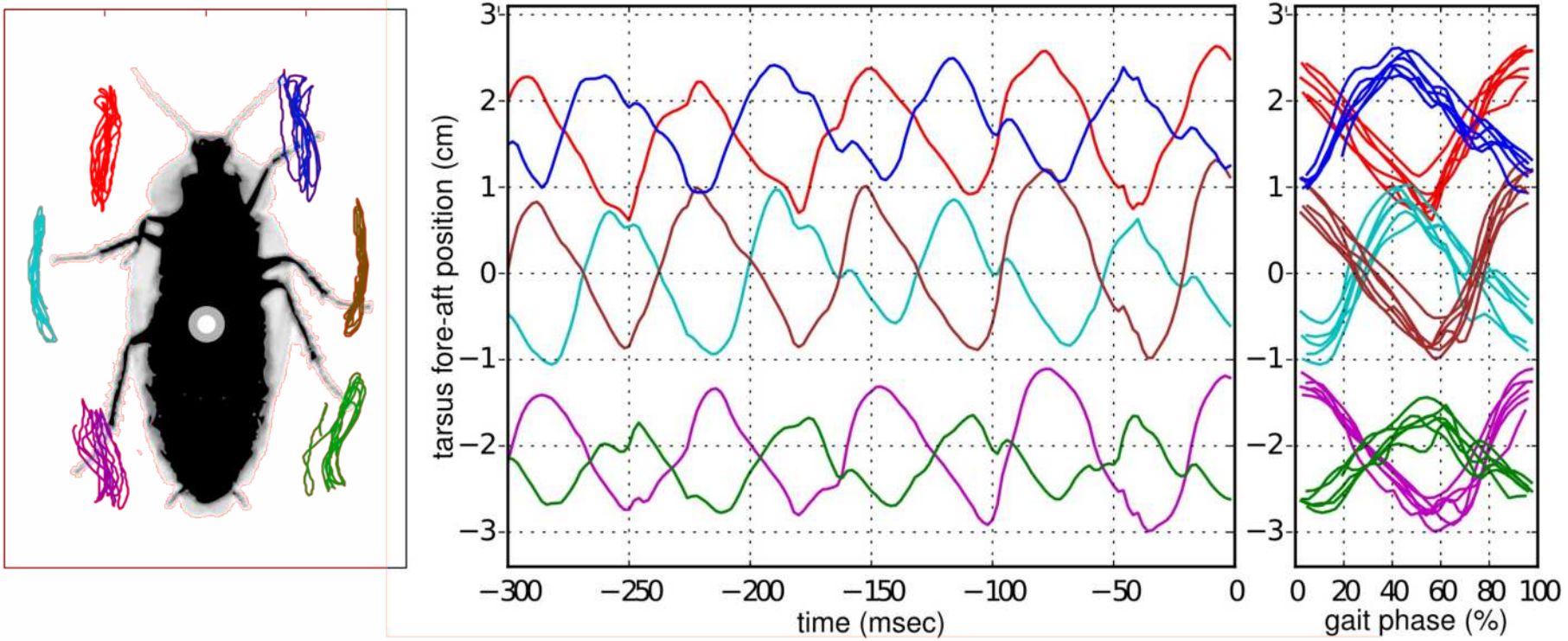
Identifying parameters and validating models

1. Reduced-order models possess unknown parameters
2. Applying nonlinear regression to piecewise-defined dynamics
3. Validating a dynamic model's response to a perturbation – the Lateral Leg Spring

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Rhythmic motion is stereotyped & low-dimensional



Revzen, Koditschek, & Full 2009; Revzen & Guckenheimer 2011

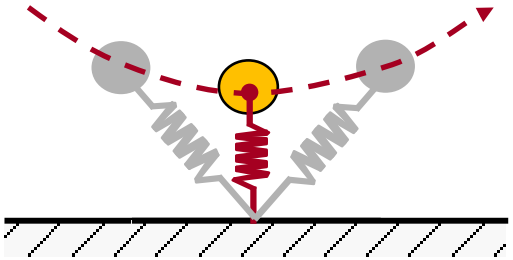
Mechanisms are partially understood

- synchronization
  Cohen, Holmes, & Rand 1982
- symmetry
  Golubitsky, Stuart, Buono, & Collins 1999
- synergy
  Ting & Macpherson 2005
- solidification
  Li, Umbanhower, Komsuoglu, Koditschek, & Goldman 2009

Gait dynamics described by reduced-order model

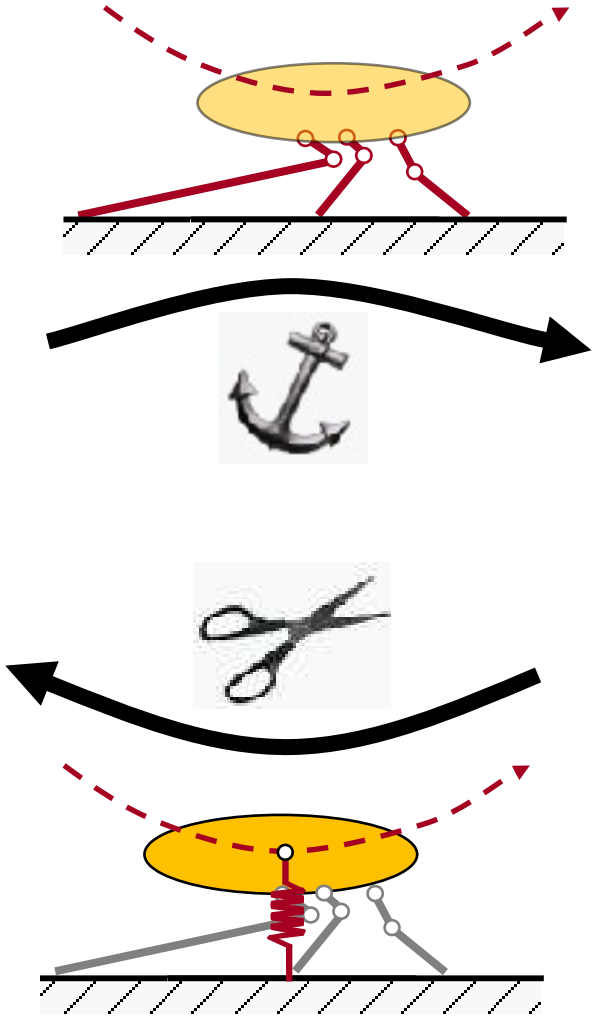
TEMPLATE

Simple model of behavior used as control target



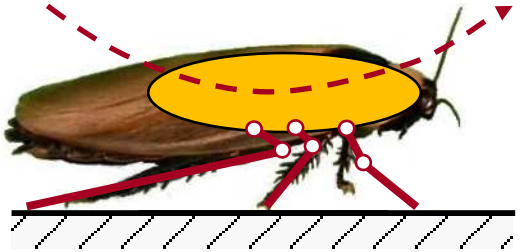
~10^1 DOF

Collapse dimensions by trimming away complexity



ANCHOR

Representative model with preferred posture



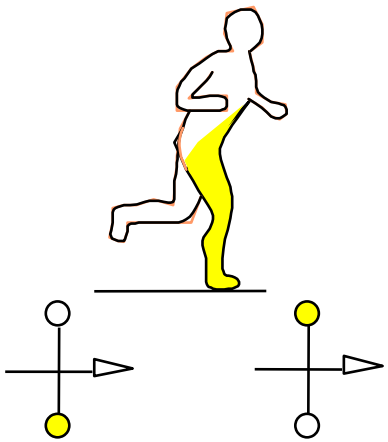
$10^?$ DOF

Reveal mechanisms by adding legs, joints, muscles

Full & Koditschek 1999

Relative leg stiffness shared across morphology, scale

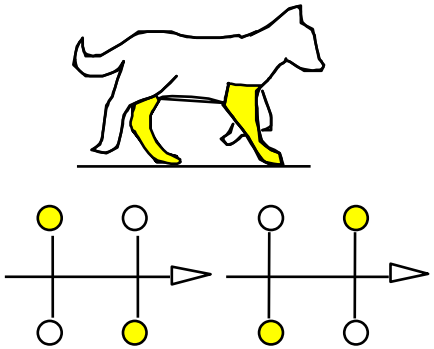
2 legs



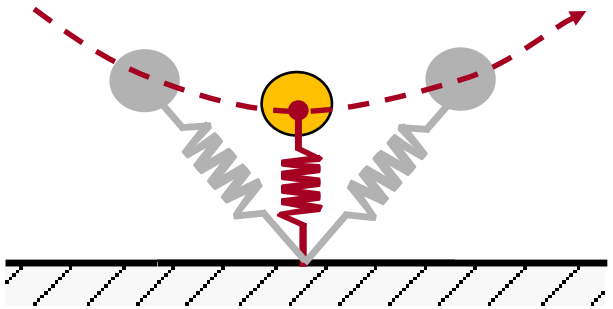
Human

Cavagn, Heglund, & Taylor 1977

4 legs



Dog



F = vertical force

m = mass

g = acceleration due to gravity

L = leg length

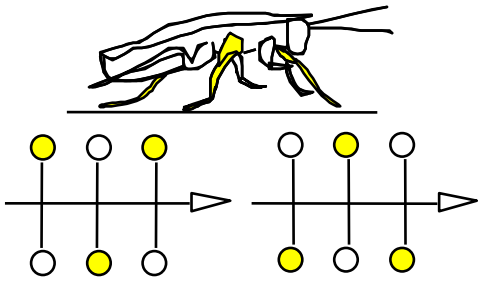
ΔL = change in leg length

n = # of legs

$$k_{rel,leg} = \frac{1}{n}\frac{F/mg}{\Delta L/L} \approx 10$$

Blickhan & Full 1993

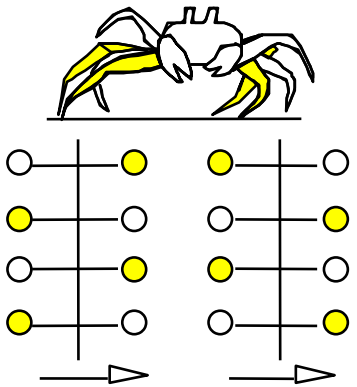
6 legs



Cockroach

Full and Tu 1990

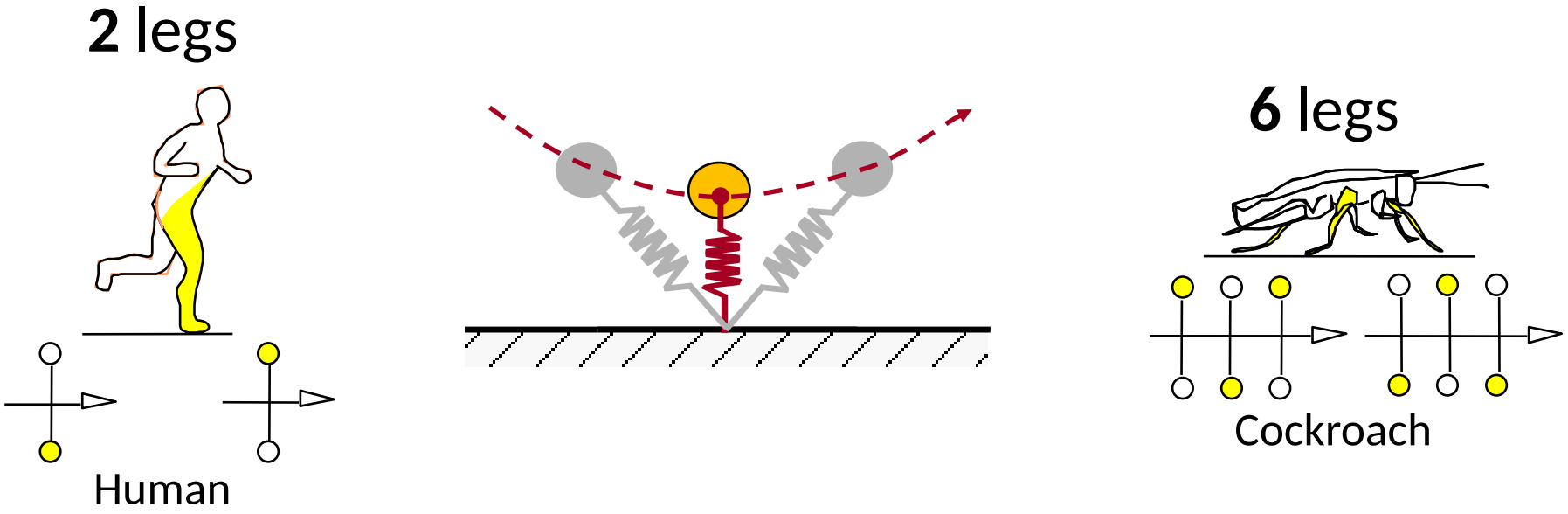
8 legs

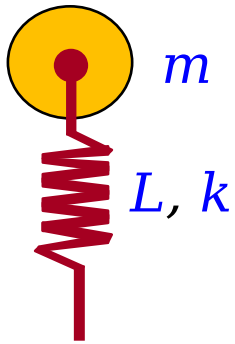


Crab

Blickhan and Full 1987

Parameter identification and model validation





- Model parameters not known *a priori*
  - leg length L and stiffness k; body mass m
- Model validity depends on parameter values
  - gait stability, parameter sensitivity, etc.

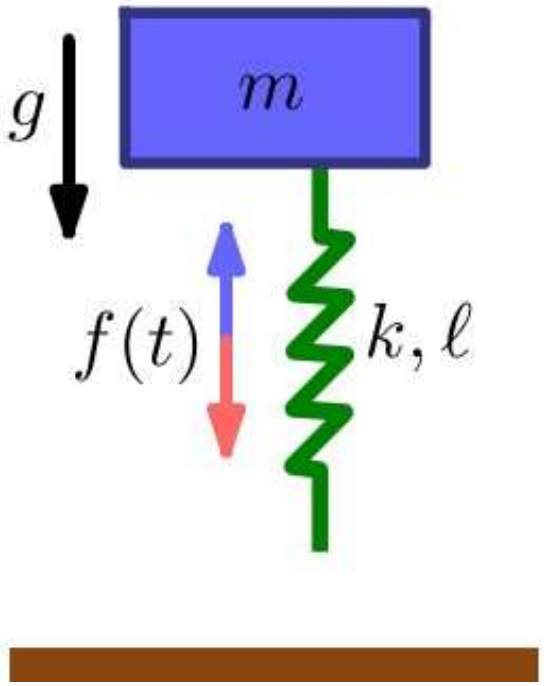
Full, Kubow, Schmitt, Holmes, & Koditschek 2002; Seipel & Holmes 2007; Srinivasan & Holmes 2008; Lipfert, Günther, Renjewski, Grimmer, & Seyfarth 2012

Need statistical tools to estimate unknown parameters from data

Identifying parameters and validating models

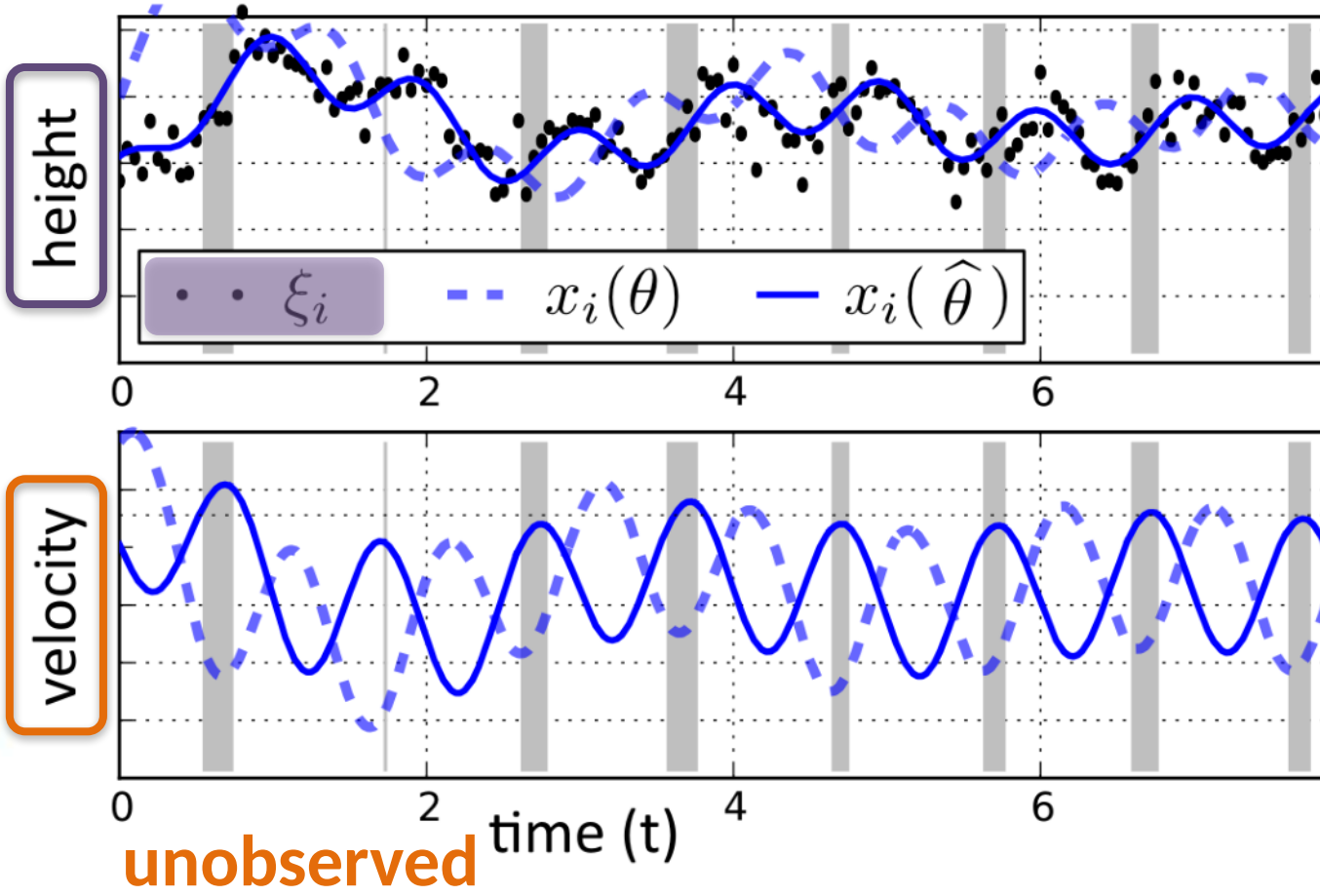
1. Reduced order models possess unknown parameters
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Parameter identification from empirical data



$\theta = (m,\ k,\ l,\ x_0,\ v_0)$

observed

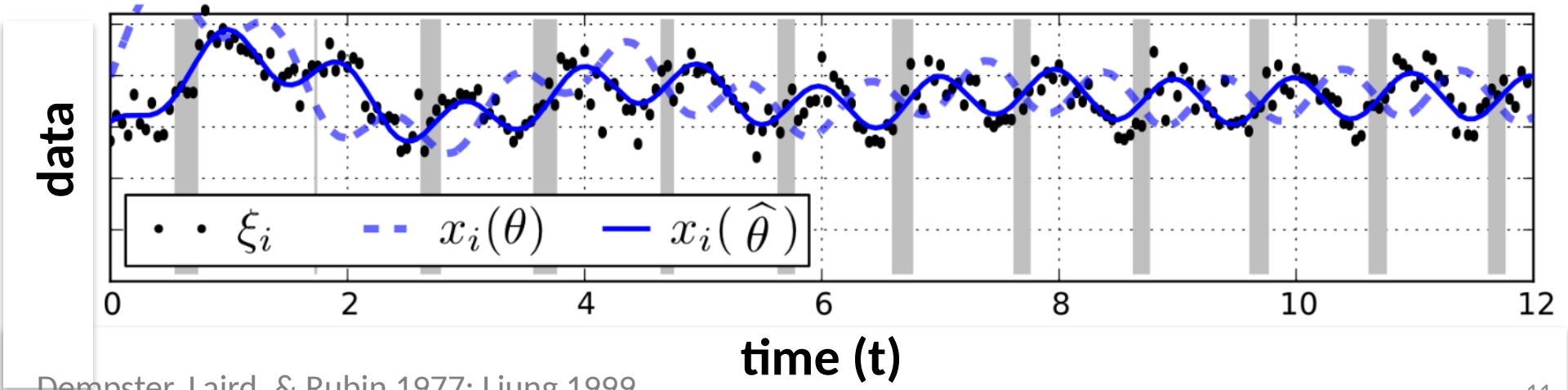


unobserved

Choose θ to make $x_i(\theta)$ close to ξ_i

Proposed technique: nonlinear regression

1. Problem setup
   - empirical time series data $\{\xi_i\}_{i=1}^N$
     (e.g. kinematic & dynamic data for body and limbs)
   - model time series data $\{x_i(\theta)\}_{i=1}^N$ as a function of initial condition θ
     (initial condition includes unknown model state & parameters)
2. Minimize prediction error $\widehat{\theta} = \arg\min_{\theta} \varepsilon\left(\{x_i(\theta)\}_{i=1}^N, \{\xi_i\}_{i=1}^N\right)$
   - (ε is, e.g., mean square error)

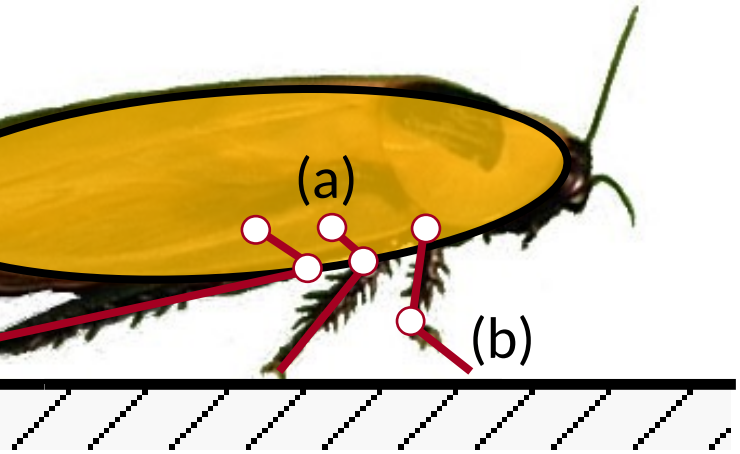


Dempster, Laird, & Rubin 1977; Ljung 1999

Problem: computationally prohibitive in general

1. Problem setup
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- Computational challenges for terrestrial locomotion:



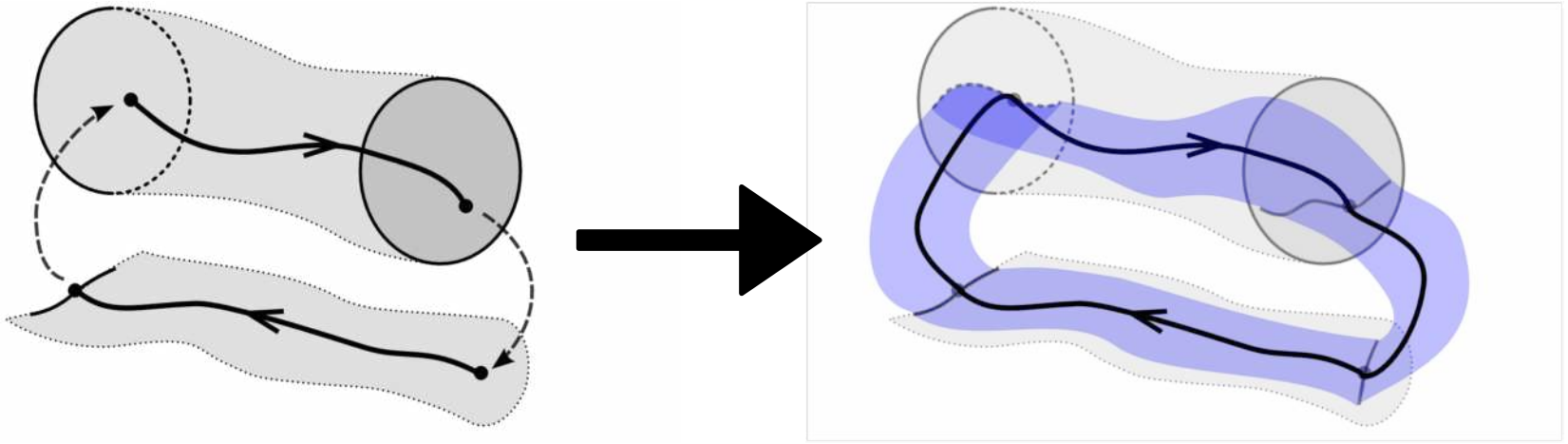
a) body-limb interaction yields nonlinear dynamics, non-convex prediction error
   - linear system ID not applicable
   - must approximate parameters

b) limb-substrate interaction introduces discontinuities in parameters
   - combinatorial (mixed-integer) optimization problem in general

Dempster, Laird, & Rubin 1977; Ljung 1999; Tugnait 1981; Paoletti, Juloski, Ferrari-Trecate, & Vidal 2007

Solution: smooth piecewise-defined dynamics

1. Problem setup
   - empirical time series data $\{\xi_i\}_{i=1}^N$
   - model time series data $\{x_i(\theta)\}_{i=1}^N$ as a function of initial condition θ
2. Minimize prediction error $\widehat{\theta} = \arg\min_{\theta} \varepsilon\left(\{x_i(\theta)\}_{i=1}^N, \{\xi_i\}_{i=1}^N\right)$



global search needed

- limited to ~10 parameters

first-order (gradient) methods apply

- can support ~100 parameters
- applicable to rhythmic gaits

Burden, Revzen, & Sastry, IEEE CDC 2011; Burden, Ohlsson, & Sastry, IFAC SysID 2012

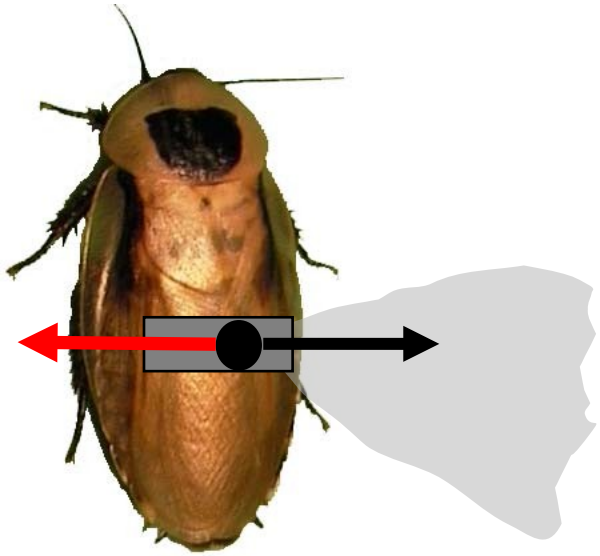
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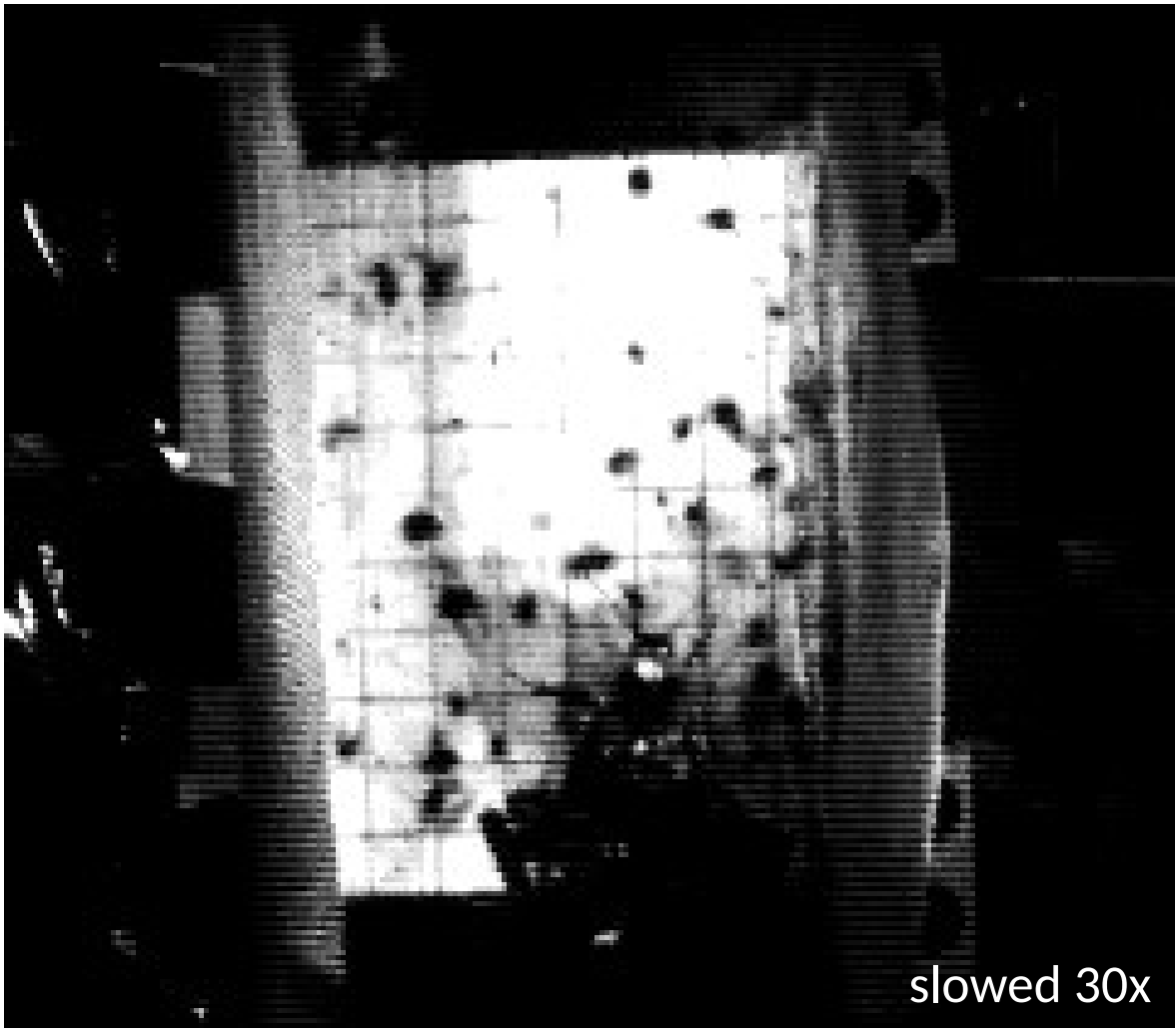
Evidence for mechanical self-stabilization

Recovery begins <13ms after perturbation

Challenges fastest neural reflexes



Rapid Impulse Perturbation Device



When does neural feedback alter recovery?

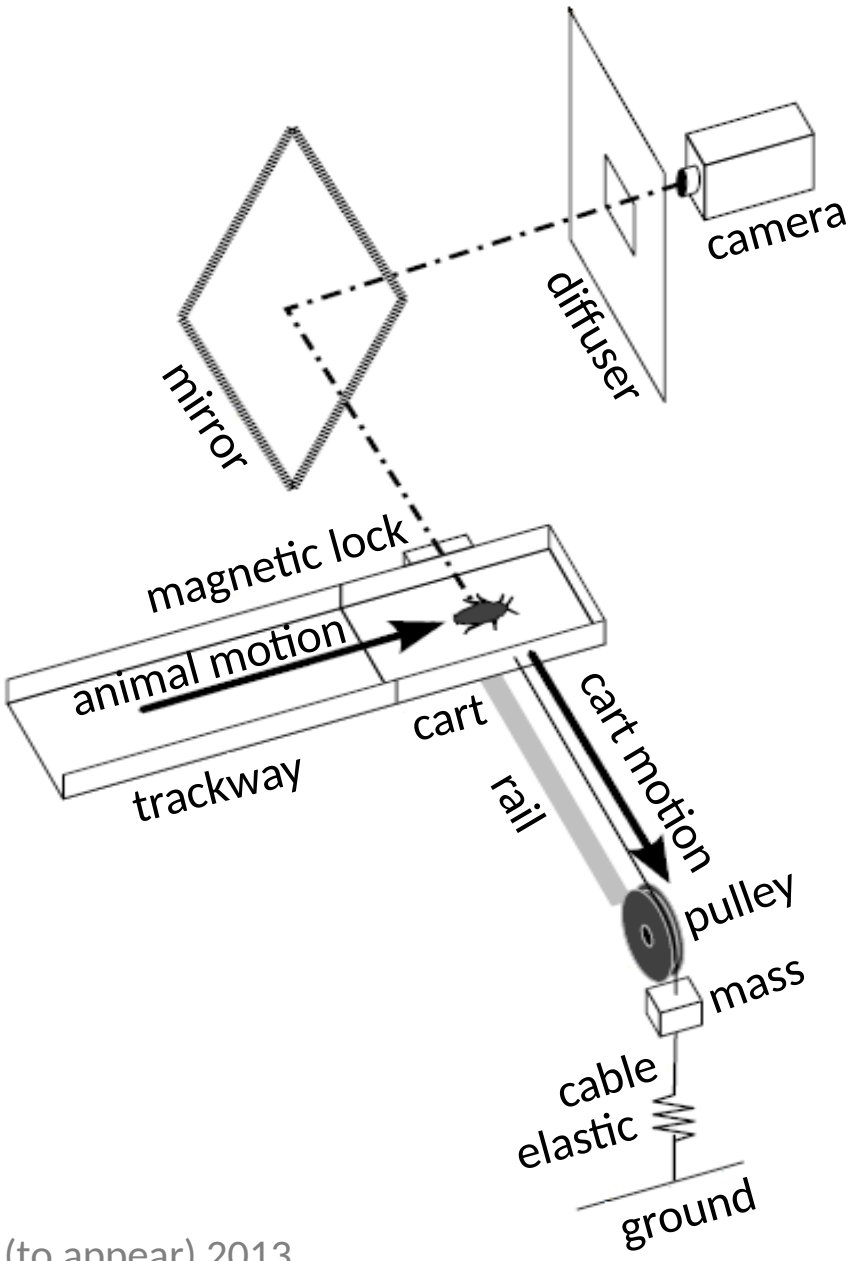
Jindrich & Full 2002

Lateral perturbation experimental setup

Platform accelerates laterally at 0.6 ± 0.1 g in a 0.1 sec interval providing a 50 ± 3 cm/sec specific impulse, then maintains velocity.

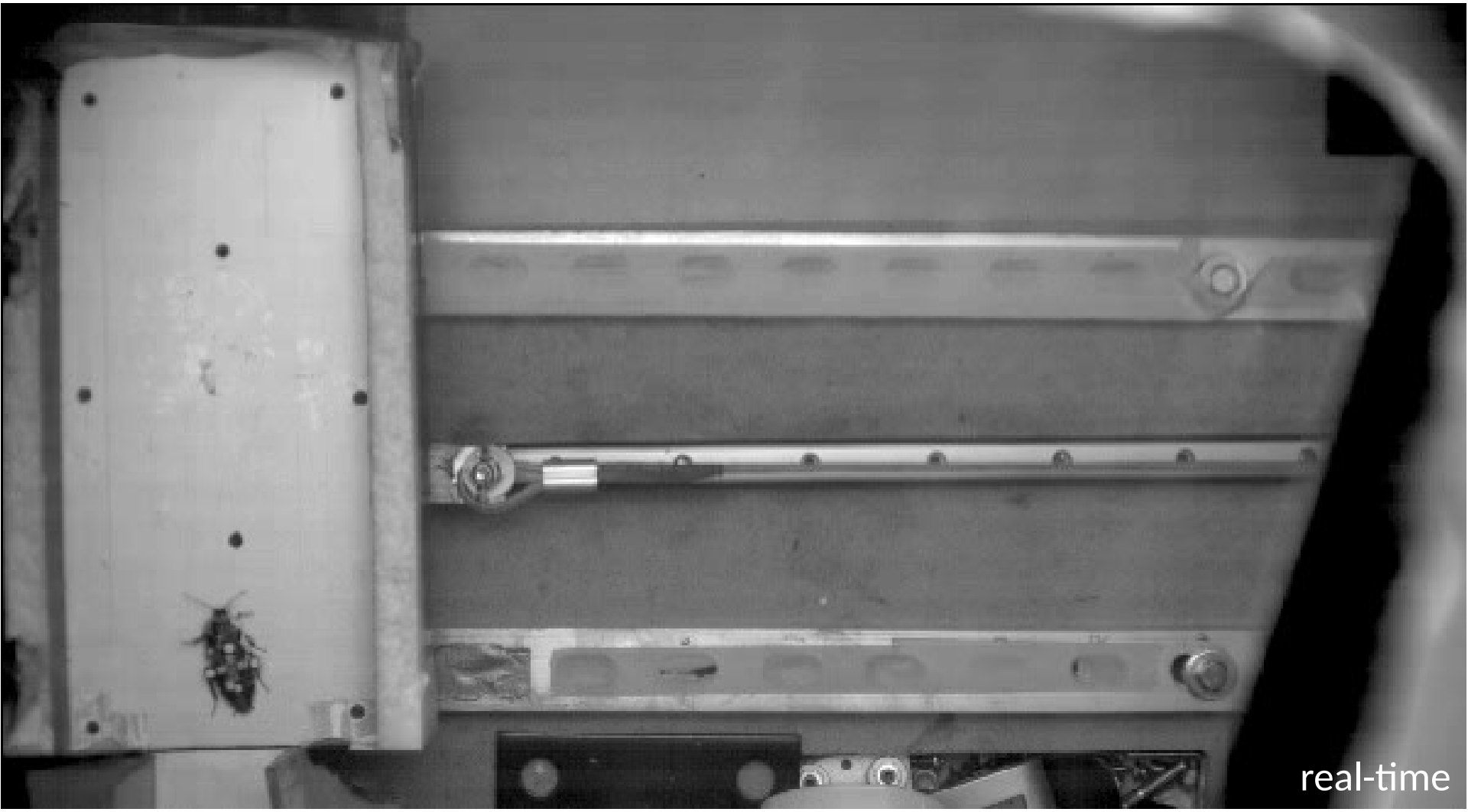
Cockroach running speed:
36 ± 8 cm/sec

Stride frequency:
12.6 ± 2.9 Hz
(~80ms per stride)

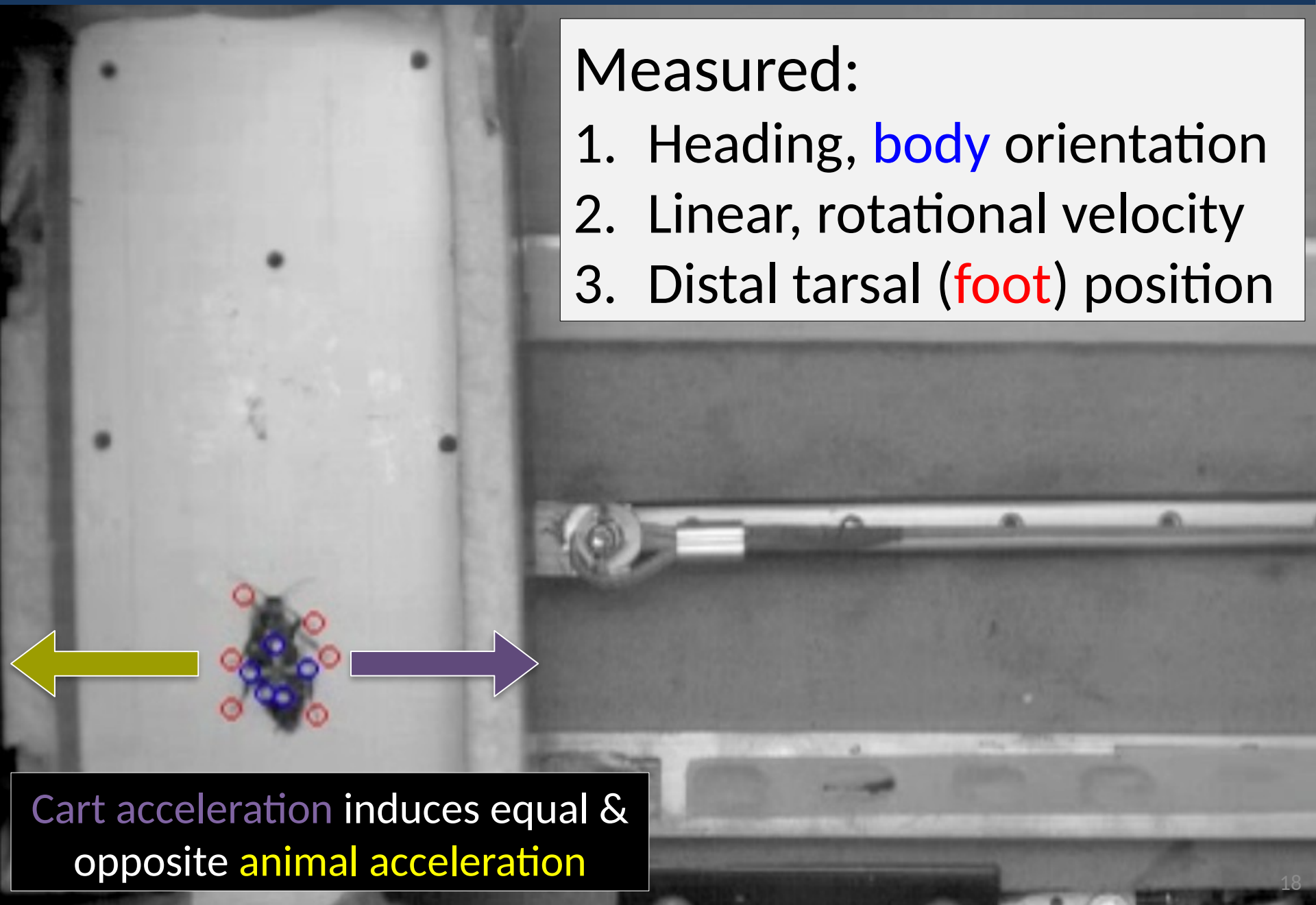


Revzen, Burden, Moore, Mongeau, & Full, Biol. Cyber. (to appear) 2013

Lateral perturbation experiment



Lateral perturbation experimental setup

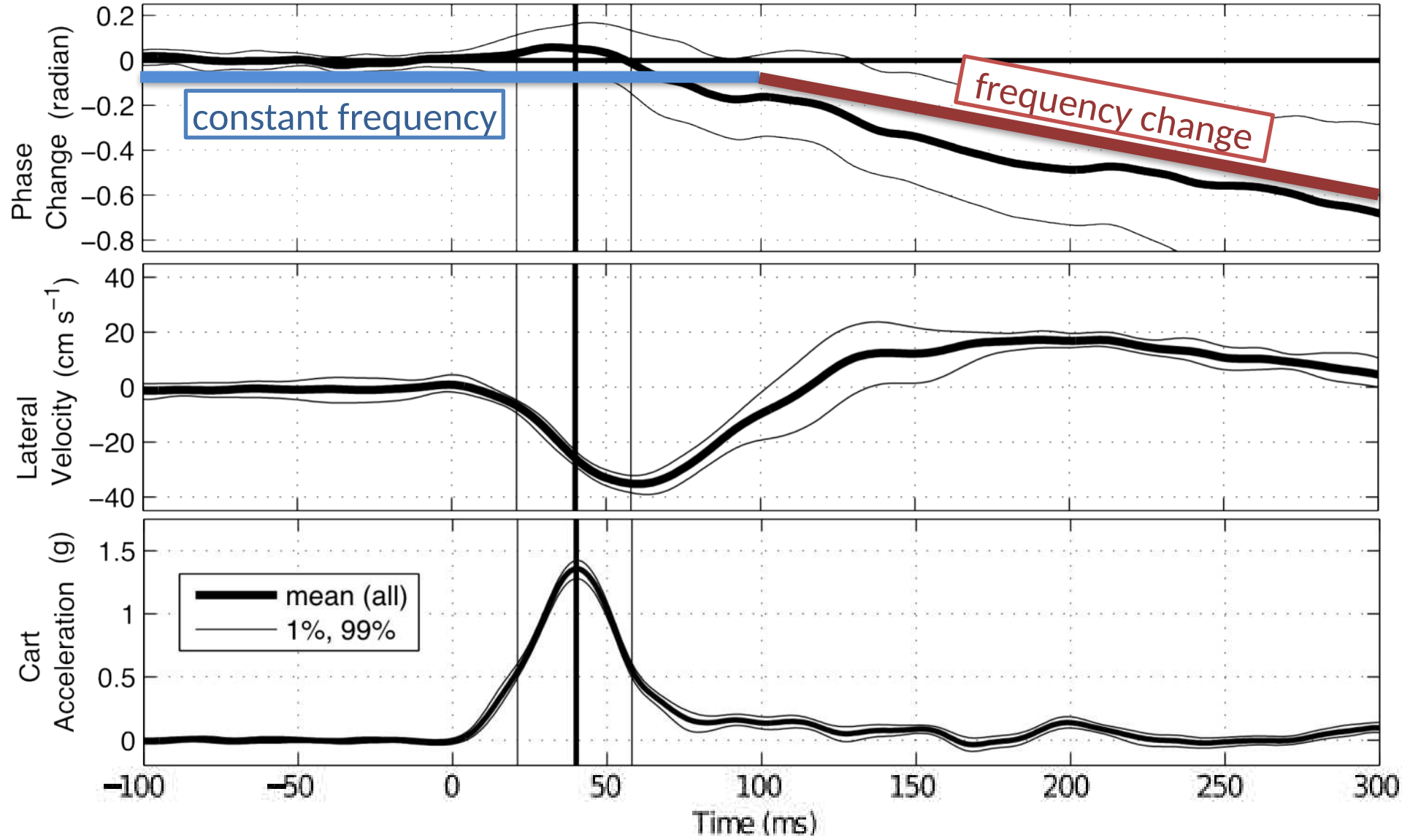


Measured:

1. Heading, body orientation
2. Linear, rotational velocity
3. Distal tarsal (foot) position

Cart acceleration induces equal & opposite animal acceleration

Lateral perturbation experimental results

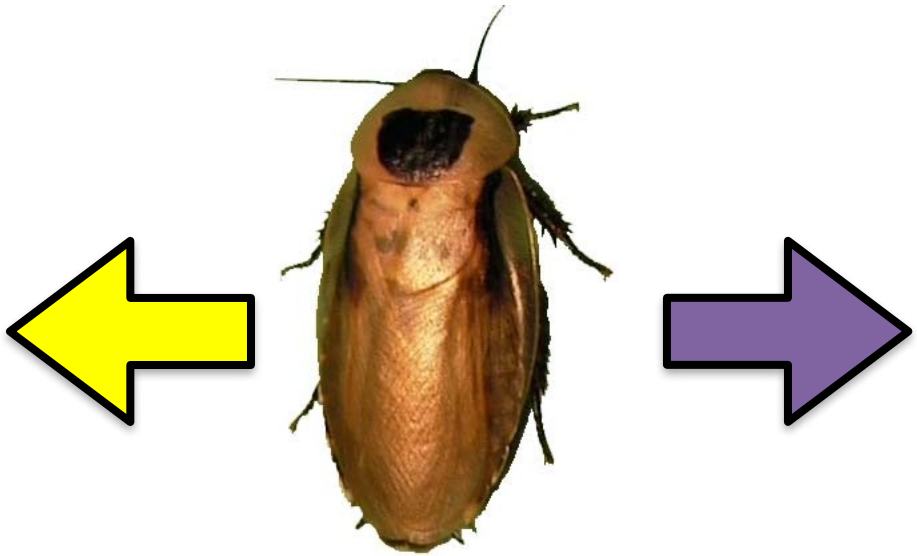


How does neural feedback alter recovery?

Revzen, Burden, Moore, Mongeau, & Full, Biol. Cyber. (to appear) 2013

Model for mechanical self-stabilization

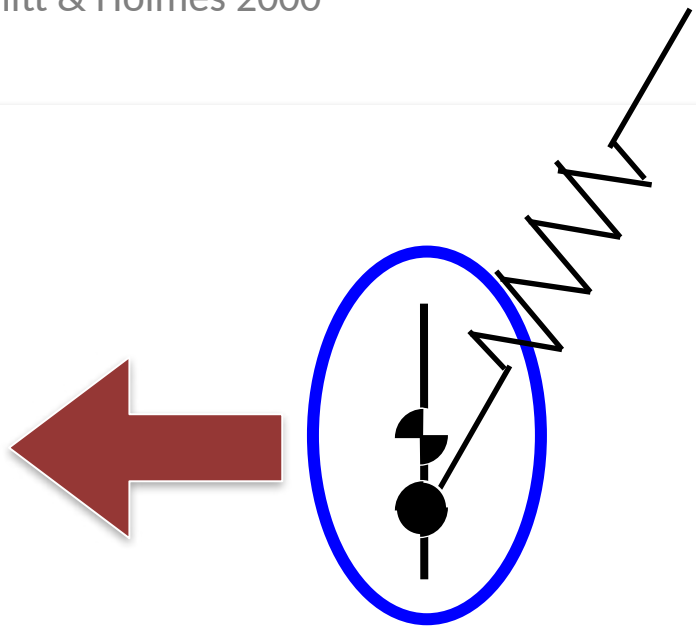
Animal



Cart acceleration induces equal & opposite animal acceleration

Lateral Leg Spring (LLS)

Schmitt & Holmes 2000

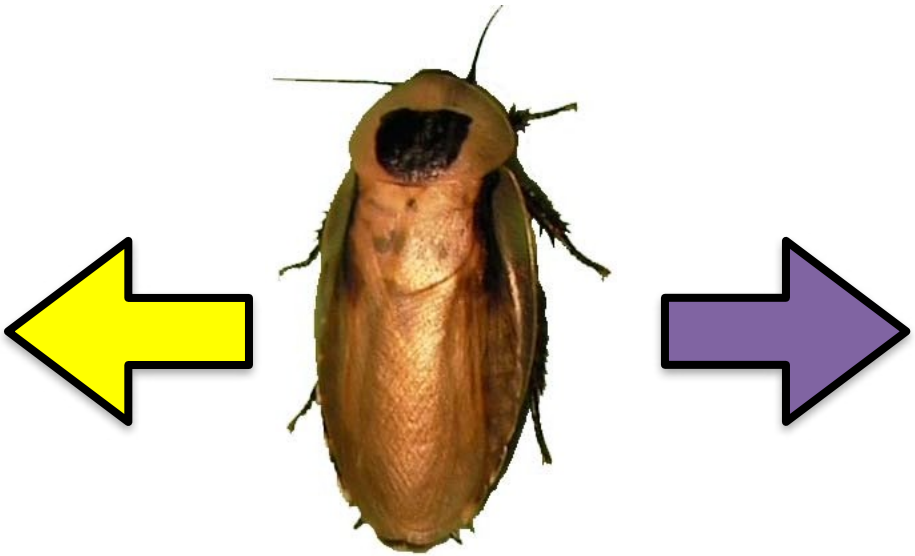


Apply measured acceleration directly to model

Quantitative predictions for purely mechanical feedback

Models for mechanical self-stabilization

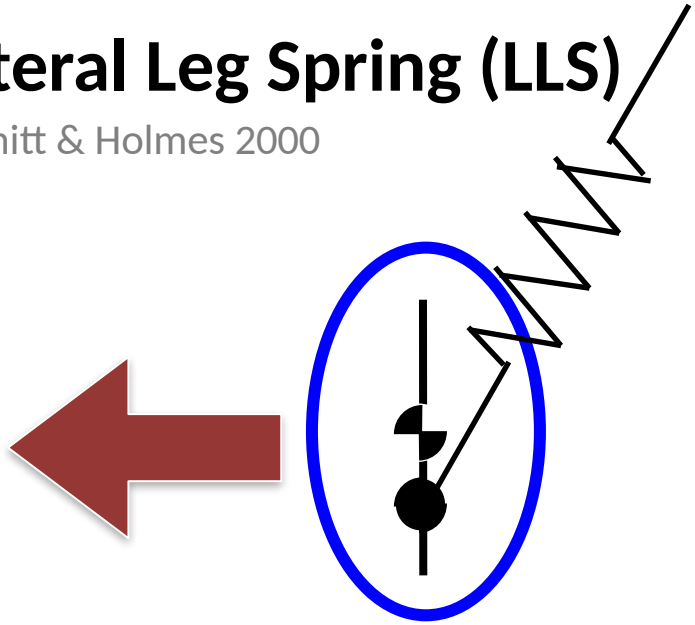
Animal



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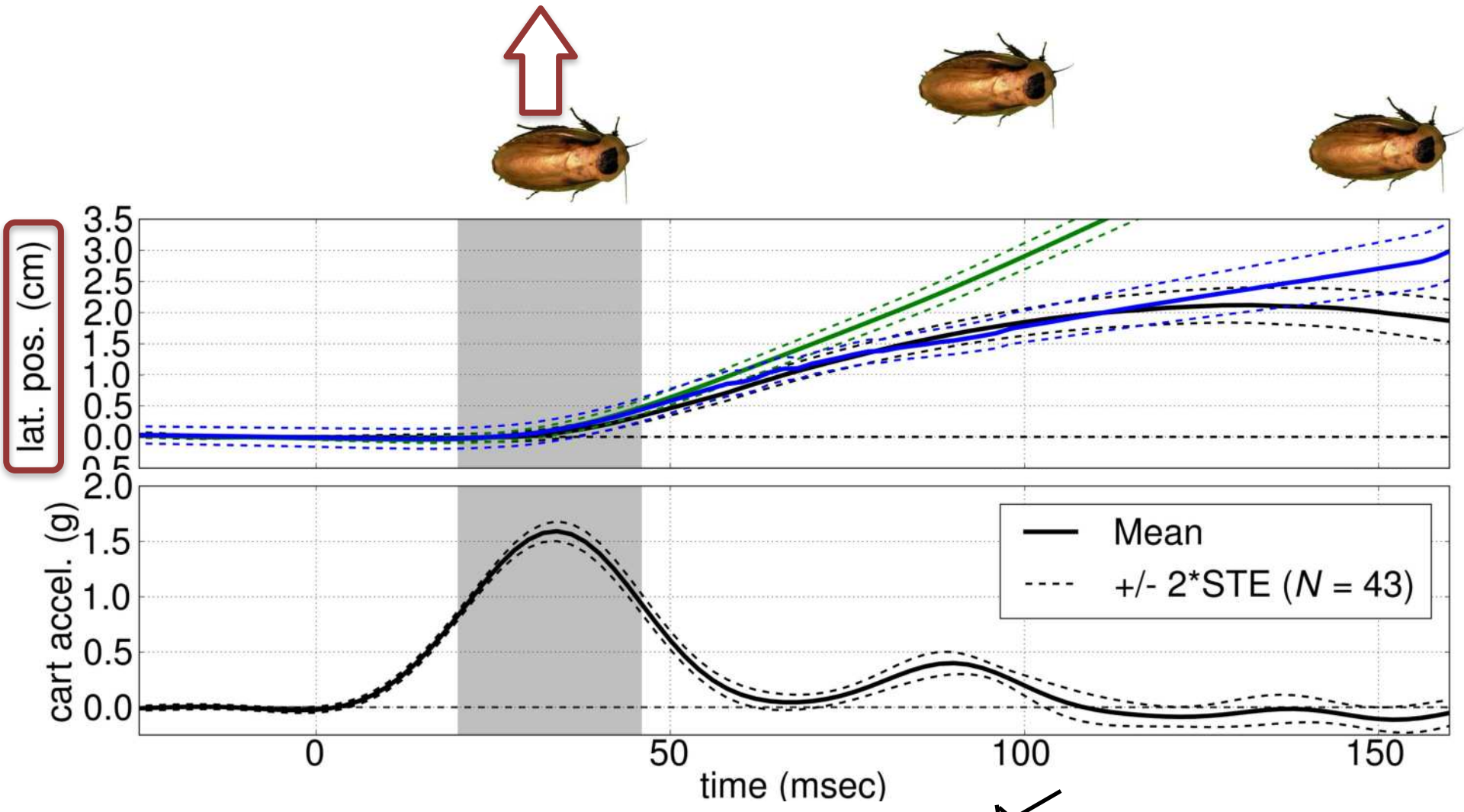


Apply measured acceleration directly to **model**



Inertial Disc

Preliminary results identifying LLS and Disc

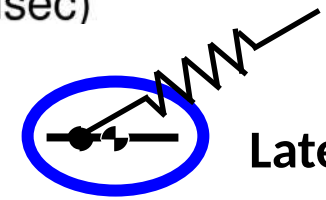




Animal

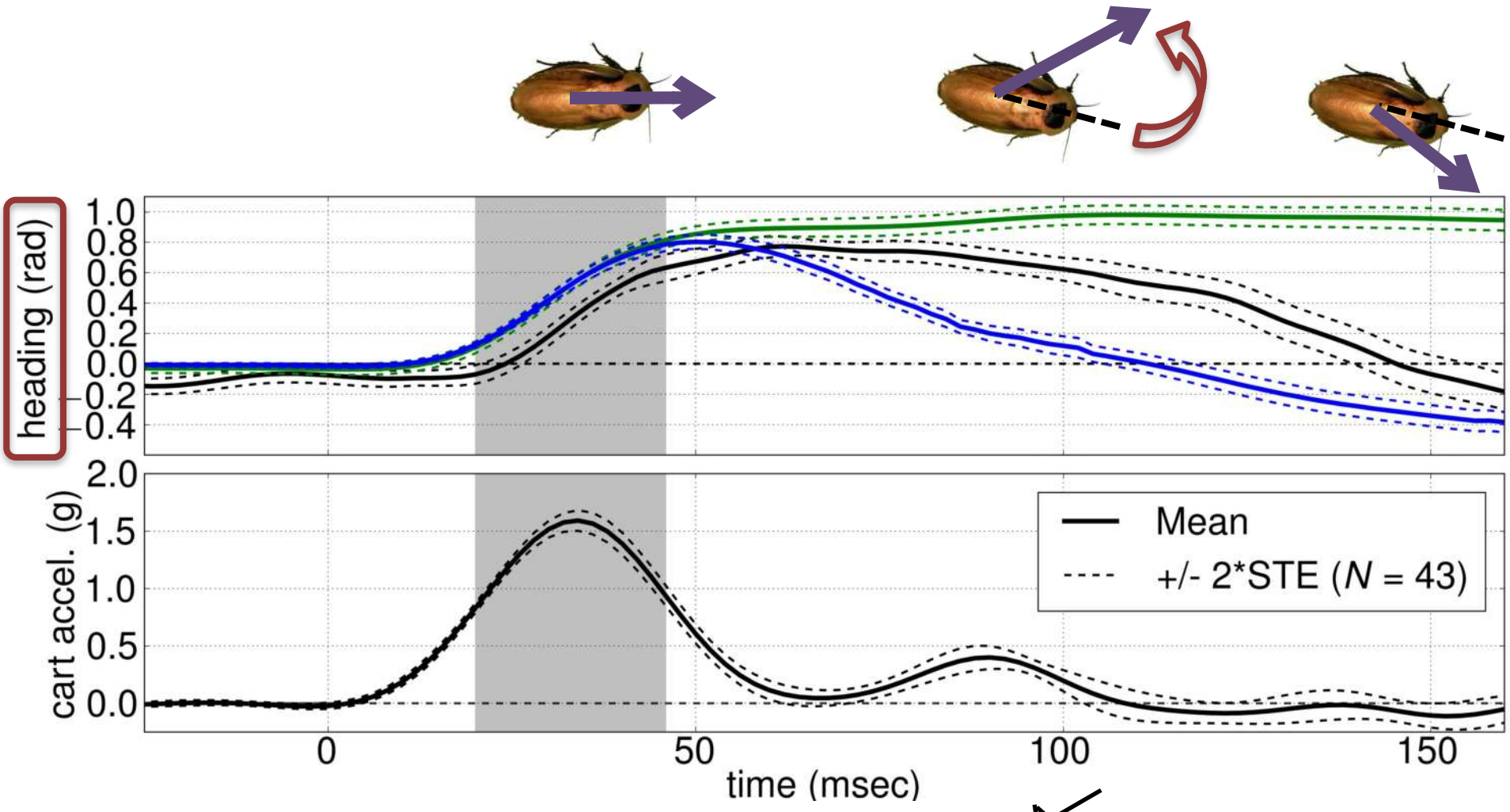


Inertial Disc



Lateral Leg Spring (LLS)

Preliminary results identifying LLS and Disc





Animal

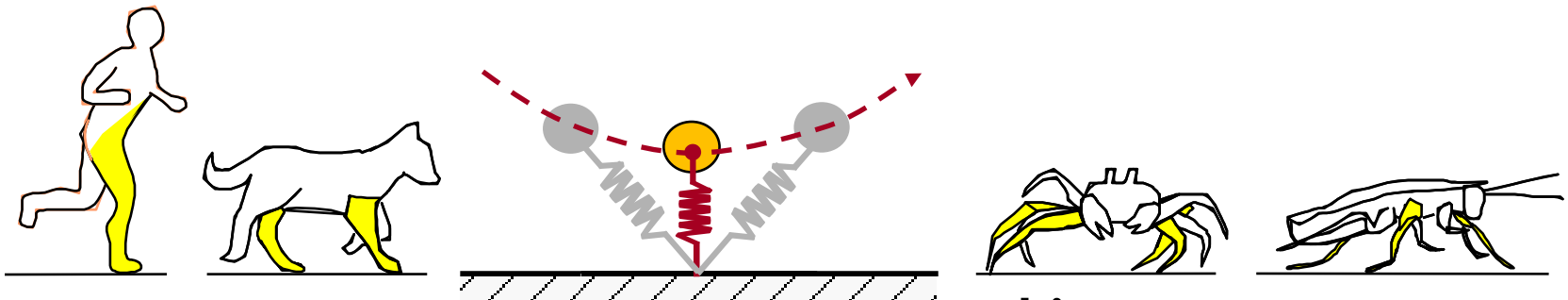
Inertial Disc

Lateral Leg Spring (LLS)

Conclusions for modeling dynamic locomotion

1. Reduced-order models must be identified & validated using data
2. Scalable algorithm for estimating parameters in piecewise-defined models using nonlinear regression
3. Validated model can provide novel quantitative predictions for experimental outcomes

Applications in comparative biomechanics



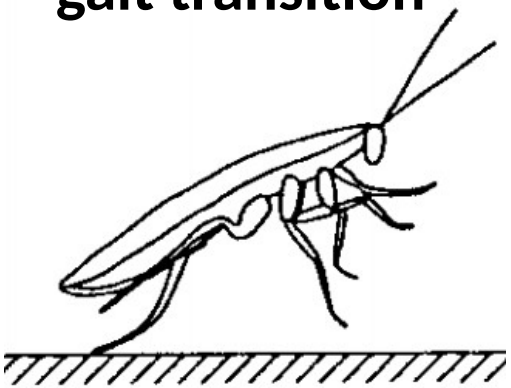
neuromechanical control architecture

limb loss & ontogeny



Pfeiffenberger & Hsieh SICB 2013

gait transition



Full & Tu 1991

dynamic maneuver



Libby, Gudenus, Harrington, & Full SICB 2013

2:40pm Imperial B

Reduced-order models support comparison across morphology & scale

Acknowledgements

Collaborators



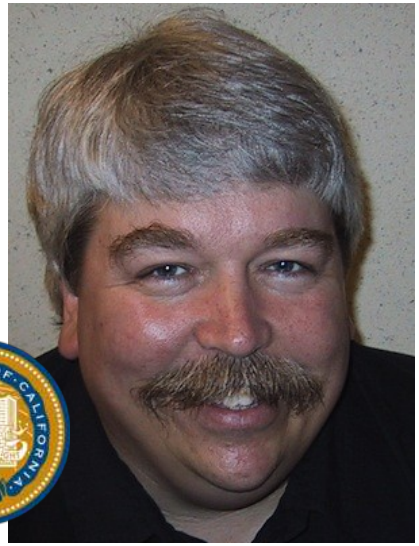
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- Biomechanics Group
- Autonomous Systems Group

Sponsors

- NSF GRF



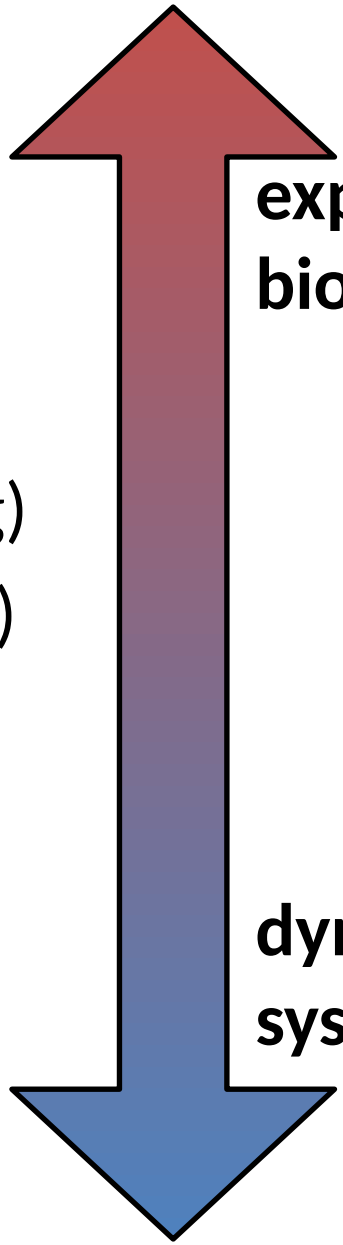
- ARL MAST



Thank you for your time!

Open problems and future directions

- choosing prediction error
- representing perturbations in reduced-order models
- model reduction (“template”-ing)
- model embedding (“anchor”-ing)
- asymptotic consistency of parameter estimates
- accurate, efficient simulation of piecewise-defined dynamics



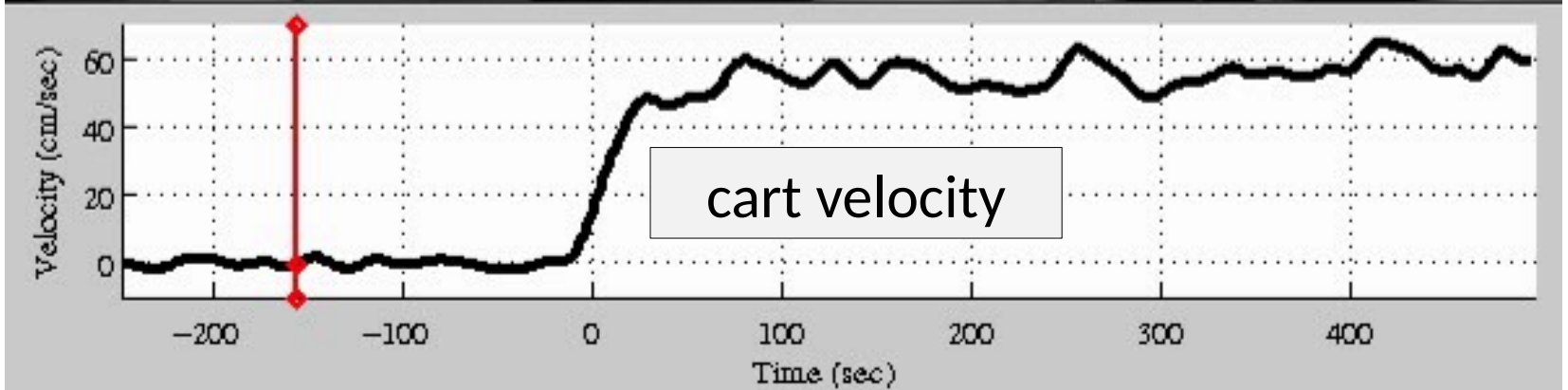
experimental biomechanics

dynamical systems theory

Burden, Ohlsson, & Sastry, IFAC SysID 2012
Burden, Revzen, & Sastry, IEEE CDC 2011
Burden, Gonzalez, Vasudevan, Bajcsy, & Sastry, IEEE CDC 2011

Lateral perturbation experiment





Lateral perturbation reference frames

