Perfect (“deadbeat”) Feed-Forward Rejection of Uncertainty in Contact Events

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Introduction

Many mechatronic or biological systems perform tasks with intermittent mechanical contact [3] – grasping or walking, for example. Dramatic changes in control authority may take place between phases [1, 2]. A perfect model is rarely available, so contacts may be made or broken at unexpected times. By defining a proper type of goal function, a manifold of state space can be defined such that trajectories confined to the manifold reject the state uncertainty introduced by the uncertainty in transition time.

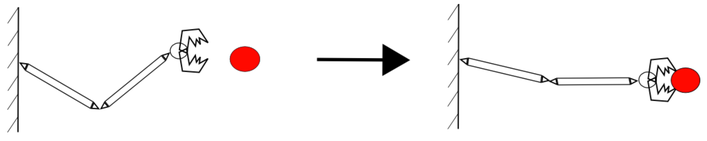

Figure 1: Load-limited actuator illustrates changes in control authority based on contact.

Objectives

1. Develop a recipe for rejecting transition uncertainty in a hybrid dynamical system.

2. The controller will exploit the freedom in control permitted in one region to enforce meeting a goal function on a Poincaré (transverse) section *S* (fig. 2).

Controller Development

The basic system structure is given as :

$$\dot{x} = \begin{cases} f_A(x,u) & x \in A, f_A : \mathrm{A} \times \mathbb{R}^k \to \mathrm{TA} \\ f_B(x) & x \in B, f_B : \mathrm{B} \to \mathrm{TB} \end{cases}$$

1. The vector fields are Lipschitz on the closures of their domains, and that the domains A, and B, and have open $\mathcal{O} \subset \mathrm{A} \cap \mathrm{B}$.
2. Control authority in A is complete, while zero control authority exists in B.
3. Define an at least continuously differentiable ‘goal’ function: $g : \mathcal{O} \times S \to \mathbb{R}^d, d < \min(\dim(B), \dim(A))$
4. Jacobian matrices of the derivative of g, $\mathbf{D}_O g$ and $\mathbf{D}_S g$, are of maximal rank, so that $\mathbf{D}g$ has rank $r > d$.

A known initial trajectory exists, i.e: $x(t_0) \in A, x(t_1) \in S,\ x^* \coloneqq x(t_*)$, $g(x(t_1)) = 0$, where x^* is the transition point that produces the desired behavior.

Given the flow of the system in B, ϕ_B, is a diffeomorphism, define $g_*(x) = g(x, \phi_B(T_B(x), x))$, where T_B is a $\mathcal{C}^1$ function giving the time-to-impact of a state to *S*.

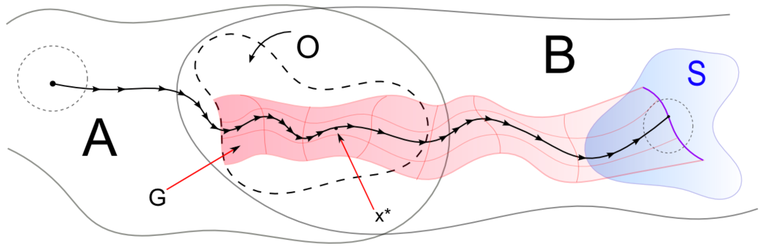

Figure 2 : System structure – A is a high-controller region, while B is a low-controller region

The level sets of g_* determine a submanifold G, $x^* \in G$. As g_* determines the eventual value of g on *S*, trajectories found only in G will satisfy the goal function regardless of the transition event. Therefore, while the a trajectory is in A, the control authority present can be used to bring a given trajectory to G. As G is a function only of the model dynamics, the manifold can be computed in advance, so that the control objective along solution $x(t)$ is :

$$\frac{d}{dt} g_*(x(t)) = \mathbf{D}g_* \dot{x} = [\mathbf{D}_O g + \mathbf{D}_S g \cdot (\mathbf{D}_x \phi_B + f_B \cdot \mathbf{D}_x T_B)]\dot{x} = 0$$

Example – Non-conservative SLIP

The method was numerically implemented on a Spring-Loaded Inverted Pendulum (SLIP - a common hybrid model of legged locomotion). We used non-conservative stance dynamics, representative of muscle models.

We developed a feed-forward control scheme that preserved relative ground height stride-to-stride without sensing the location of the ground.

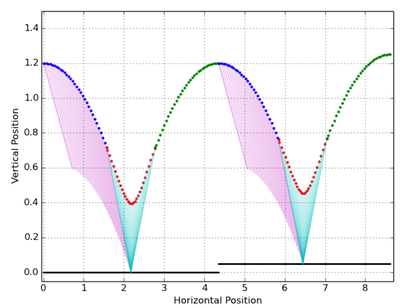

Figure 3 : Model of SLIP – leg angle is varying with time while in air, as in internal damping coefficient

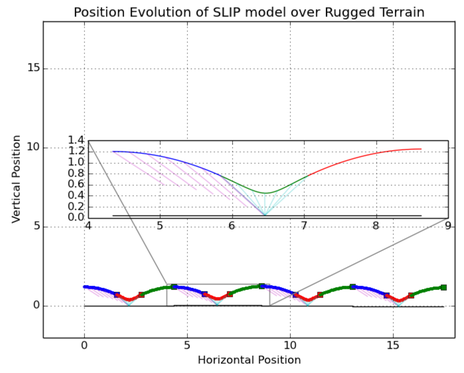

Figure 4 : Position of hopper COM as a function of time with controller implemented. Ground height variation L = [-0.05, 0.05]. As the control scheme is inherently a local result, arbitrarily large alterations in the ground height will be necessarily be rejectable with the suggested scheme.

Conclusions

The apparently feedback-orientated task of rejecting transition uncertainty can instead be accomplished confining trajectories to a pre-defined manifold in a feedforward manner. While control freedom in A could be utilized to achieve the objective near x^*, given a system defined so that its open-loop dynamics naturally complied with the manifold, despite no apparent controller, such a system would exhibit desirable stability and tracking properties.

References

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