Phase driven models of unperturbed locomotion

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Objective

We would like to be able to predict the leg movements of dogs, the beat of a heart and the oscillations of electric fields in the brain; cyclic biological behaviours.

In the long term, for deterministic systems, a single co-ordinate, asymptotic phase, captures almost all of the variance in limit cycle oscillators.

For stochastic systems no equivalent theory exists. We develop a notion of phase as a closed differential 1-form which is equivalent to asymptotic phase in the deterministic case.

Using this 'temporal 1-form' we build a better data-driven estimate of asymptotic phase. We train the temporal 1-form on training data (from both stochastic simulations and guineafowl) of (potentially partial) trajectories of the motion of interest.

Theory

Look for a one-form ω, satisfying the requirement

$$\omega \cdot f = C$$

where f is a tangent vector from our training trajectories and C is a constant. With N observations enumerated by i, and our form adjustable by parameters p, we proceed by solving

$$\min_p \sum_{i=1}^{N} \left(\omega\left(p\right) \cdot f_i - C\right)^2$$

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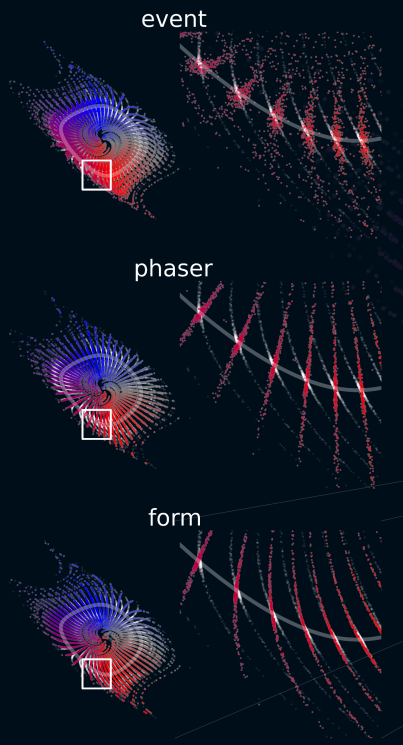
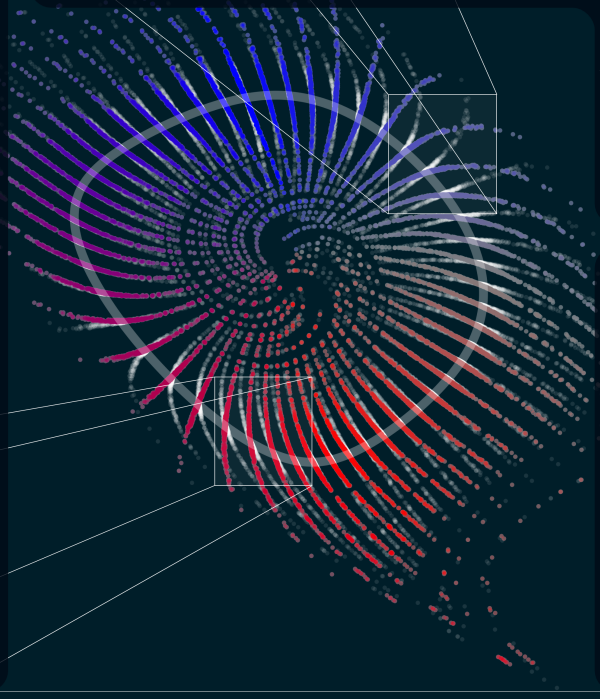
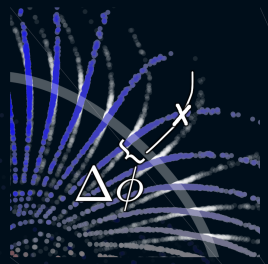
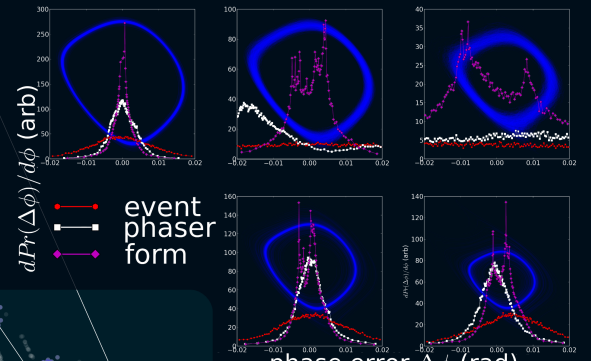
Results

Estimated isochrones in color; 'event' derived by interpolating time between consecutive characteristic events (cf. heel strike); 'phaser' from Revzen(2008); 'form' derived by estimating the temporal 1-form. Limit cycle is the grey loop, ground truth isochrone of the deterministic system are in white.

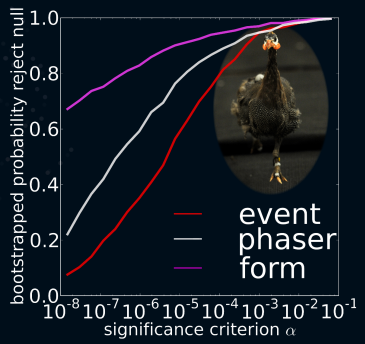








Partial probability plot for error in residual phase. Form phase (purple) does better than phaser (white) and event based phase (red). The blue trajectories in the background indicate the test conditions.



Registering using form phase enhances our statistical power. Here we look if midstance forces in guineafowl increase with speed.

Guineafowl Image: Jim Usherwood

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Conclusion

Form phase is a superior method of estimating asymptotic phase, performing well away from the limit cycle, under a variety of different noise conditions, and with simulated and animal data.