Optimizing Gaits for Coverage on Lie Groups

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***Abstract*— Individual performance criteria, such as robustness and efficiency, often drive behavior selection in mobile robotic platforms. However, navigational capabilities can rely heavily on the coordination of all behaviors available to the robot, rather than the performance of one behavior. Here, we studied how a collection of availabile gaits (cyclic internal motions) can be related to the variability and density of external motions that a system can achieve. We attempted to capture this capability, termed "coverage", with an associative fast-to-compute cost function that can be used to simulataneously optimize a collection of gaits. The cost function can be weighted such that users may specify the importance of various classes of external motions. A resulting optimization framework was used to maximize the coverage of three gaits of a 3-link Purcell swimmer on the space of planar rigid body motions.**

I. Introduction

We assume that the configuration space $\mathcal{Q}$ of our moving animal or robot can be factored as a product of a shape $\mathcal{S}$ and a (generalized) position $\mathcal{G}$. This generalized position is a Lie group, typically a sub-group of the rigid body motions $\mathsf{SE}(3)$. Legged systems typically produce motion using "gaits", which we take to mean periodic changes in shape that produce a predictable body motion, i.e. a gait b is a function $\gamma_b : \mathsf{S}^1 \to \mathcal{S}$ that produces a body motion $M_b \in \mathcal{G}$. Given a finite selection of gaits and a means for switching between them, a planner can produce any motion that corresponds to a word in the group elements those gaits generate, e.g. with gaits γ_a and γ_b one can produce the motions $I, M_a, M_b, M_a^2, M_aM_b, M_b^2, M_bM_a, M_a^3, \ldots$ In this work we studied the problem of generating "coverage" of $\mathcal{G}$ with a small collection of gaits.

A. Specifying the loss function

As conventionally practized, a motion planner is given some parameters x, a means to generate motions $M(x) \in \mathcal{G}$, and some loss function which it will minimize. We take the loss function $\tilde{\eta} : \mathcal{G} \to \mathbb{R}^+$ to be purely a function of endpoint. We will assume that the loss function is written relative to some desired goal position G of the motion, and define a relative goal function on the Lie Algebra $\eta(\xi) := \tilde{\eta}(\mathrm{e}^{\xi}G)$. What remains is to optimize x with respect to the loss function $x \mapsto \eta \circ \log(M(x)G^{-1})$, the difficulty of which depends on the map $M(\cdot)$ and the group logarithm. A left invariant distance metric on $SE(2)$ and $SE(3)$ provides practical implementation of η[2].

We set up our optimization as follows: let $\mathbf{G} := \{G_i\}_{i=1}^n$ be a set of goal motions, and let $\mathbf{W} := \{w_i\}_{i=1}^n \subset \mathbb{R}^+$ be importance weights associated with these goals. Let $\mathbf{M} := \{M_j\}_{j=1}^m$ be a set of achievable motions. We define our coverage cost $h(\mathbf{M}) := \sum_i w_i \min_j \eta \circ \log(M_j G_i^{-1})$. We further define the "k-word coverage cost" $h_k(\mathbf{M})$ as the cost of the set of words of k or fewer elements of $\mathbf{M}$.

B. Connecting gait and motion

Given a gait γ_b, the body frame motion M_b it produces could, in principle, be a function of the initial point in the gait cycle, and the speed with which this cycle is executed. For systems where momentum is dominated by friction or constraints, this is not the case. In those systems there exists a map $A(s) : s \in \mathcal{S} \mapsto \mathsf{L}(\mathsf{T}_s, \mathfrak{g})$ taking shapes to linear maps from shape velocities to the Lie Algebra $\mathfrak{g}$ of $\mathcal{G}$. This leads to the "reconstruction equation" $\dot{g} = L_g A(s)\dot{s}$ where $L_g : \mathfrak{g} = \mathsf{T}_e \to \mathsf{T}_g$ is the lifted left action of the group element g. A well known example of such systems are low Reynolds number swimmers [1]; however, we have recently accumulated evidence that this theory applies to multi-legged locomotion [3]. The function $A(\cdot)$ connects gaits, as body shape loops, to the motion they induce, called the "holonomy" of the loop. Any path in the kernel distribution of $A(\cdot)$ can connect such loops to one-another without introducing additional motion.

II. Results

To illustrate the approach, we optimized three gaits for a 3-link Purcell swimming model to provide coverage of a portion of $\mathsf{SE}(2)$ surrounding the identity using their $h_3(\cdot)$ cost. Results are illustrated in Figure 1.

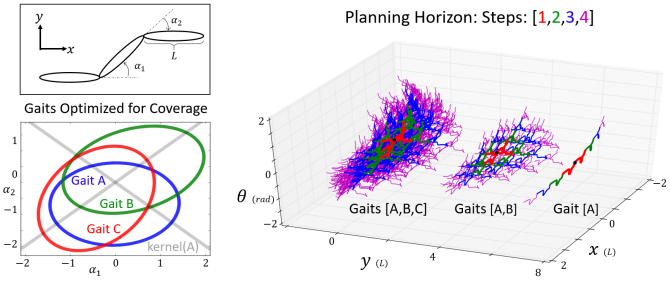


Fig. 1. We constructed G from randomly sampling 100 points of a unit ball on $SE(2)$. Ten iterations of gradient descent on the loss function (seeded with a unit circle gait), resulted in the gaits (bottom left) for the 3-link Purcell swimmer (top left). Gaits can be connected along the kernel of $A(\cdot)$ (grey), preserving the exactness of the four step planner (right). The two and one gait planners are shifted in y by 5L and 7L respectively.

References

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[2] Frank C Park. Distance metrics on the rigid-body motions with applications to mechanism design. *Journal of Mechanical Design*, 117(1):48–54, 1995.

[3] Dan Zhao, Taesung Heo, and Shai Revzen. Modeling multilegged locomotion: the friction dominated case. In *APS Meeting Abstracts*, 2018.