Is Legged Locomotion Almost Smooth?

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Commonly, legged locomotion is modeled using non-smooth (hybrid) dynamical systems. I have previously shown that many of the forms of non-smoothness that appear in legged systems are rectifiable to the smooth case. Here I will review recent progress in modeling such systems: algorithmic improvements in computing perturbation models of multi-contact problems, extensions of the reconstruction equation from geometric mechanics to the non-smooth case, and extensions of Koopman operator methods to function spaces that contain non-smooth functions.

Background

Most investigators who attempt to mathematically model legged locomotion do so with "hybrid systems". These are dynamical systems whose flow comprises a finite number of smooth parts, such that the boundary between those parts is a manifold (the "guard"). Typically each part represents a distinct set of contacts governing the motion, and the guards represent the conditions under which contacts are formed or broken.

Because animals and robots seem to utilize periodic motions to move with legs, I have historically analyzed these systems from the perspective of (hyperbolically stable) oscillator theory. This approach allowed us to show [1] that for legged systems whose contacts are built or broken one at a time, the dynamics collapse in a finite number of cycles to a "smoothable" sub-manifold. For legged systems with multiple legs that form or break contacts with near simultaneity, we have shown [2] that these systems are also conjugate to a smooth system (a flowbox), and that their flows are piecewise smooth and continuous (PCr). We have also shown how to compute the stability properties of the trajectories that go through the simultaneous contact event [3].

Together, these results (reviewed in [4]) show that the mathematical issues raised by non-smoothness of legged systems can often be resolved, leading to results very similar to those appearing in the smooth case. Here I review several recent advances in our techniques for modeling and analysis of the kind of hybrid systems that appear in legged locomotion models.

Results

Orbital Stability of Event Selected Systems: "Event Selected Systems" are a class of hybrid systems that appear in multi-legged locomotion models [2].

Our results for event selected systems allow us to write perturbation solutions for trajectories passing through the intersection of guards. In other words, we can directly discuss the stability of gaits where multiple legs hit the ground at once, or the stability of gasps where multiple fingers contact the object at once. I will summarize recent results showing how to achieve super-exponential improvements in the evaluation of the Bouligand derivatives that appear in these problems.

Reconstruction Equation for Non-Smooth Systems: One of the great strengths of geometric mechanics is the "Reconstruction Equation" (RE) which supplants the need for the more elaborate second-order equations of Newtonian mechanics. As typically defined, the RE is formulated in terms of derivatives and is therefore problematic to apply to legged locomotion. I will present an elementary argument showing that the RE extends to any system governed by a continuous flow which is "mostly" invertible -- implying that it applies to legged systems, and even those legged systems which undergo plastic collisions.

Koopman Eigenfunctions for B-Differentiable Systems: Recently, control theorists have had a growing interest in the operator theoretic methods of Koopman theory [5], which suggest new approaches for linearizing the dynamics of non-linear systems. These methods rely on the ability to find "Koopman Eigenfunctions" - observables of the dynamics which evolve exponentially in time. Unfortunately, conventional results in Koopman theory only guarantee uniqueness of these observables and their associated spectrum for C-infinity smooth systems.

I will summarize some new results proving that the Bouligand differentiable systems that arise from legged locomotion possess nearly the same uniqueness properties as smooth systems, opening the door to applying Koopman methods to the control of legged locomotion.

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