Bridging Walking and Slithering – Stokesian Locomotion

Shai Revzen*[1], Dan Zhao[1], Glenna Clifton[2], Brian Bittner[3], Nick Gravish[4]

* corresponding author shrevzen@umich.edu 1 U. Michigan; 2 U. C. Irvine; 3 JHU Applied Physics Lab; 4 U.C. San-Diego

"Stokesian Locomotion" defined

Originally formulated as a model of locomotion in a low Reynolds number "Stokes flow" fluid, we use this term for locomotion governed by an equation of the form:

$$g^{-1}\dot{g} = A(r)\dot{r}$$

Where $g \in \mathrm{SE}(3)$ is the location of the body frame in the world, and $r \in \mathcal{Q}$ is the body shape, given by some location independent parameters such as angles.

#1 RESULT: Almost all legged locomotion is Stokesian

When a foot is grounded, i.e. not slipping and detecting all 3 possible rotational motions in its shape (Fig. 1 [A]) there is a function mapping shape to body frame location $g = F_{\mathrm{COM}}(r)$ and following from that a precise Stokesian model follows. The motion constraints are governed by a true "whole" function, hence called "holonomic". No actual locomotion is possible.

To move, animals must switch contact feet, producing piecewise holonomic motion (Fig. 1 [B]). While the mapping from shape to body frame has jumps, a Stokesian representation still exists (provided contact state is a function of body shape), producing a continuous body frame location.

#2 DISCOVERY: Multi-legged slipping has a Stokesian model

Using a data-driven Stokesian model of the form in Fig. 1 [C], we could obtain good predictions of the motion of the BigANT robot [?] at various speeds and turning rates (Fig. 2).

When momentum decays in a short time ε motion is still close to Stokesian, as shown in [?] (Fig.1 [D])

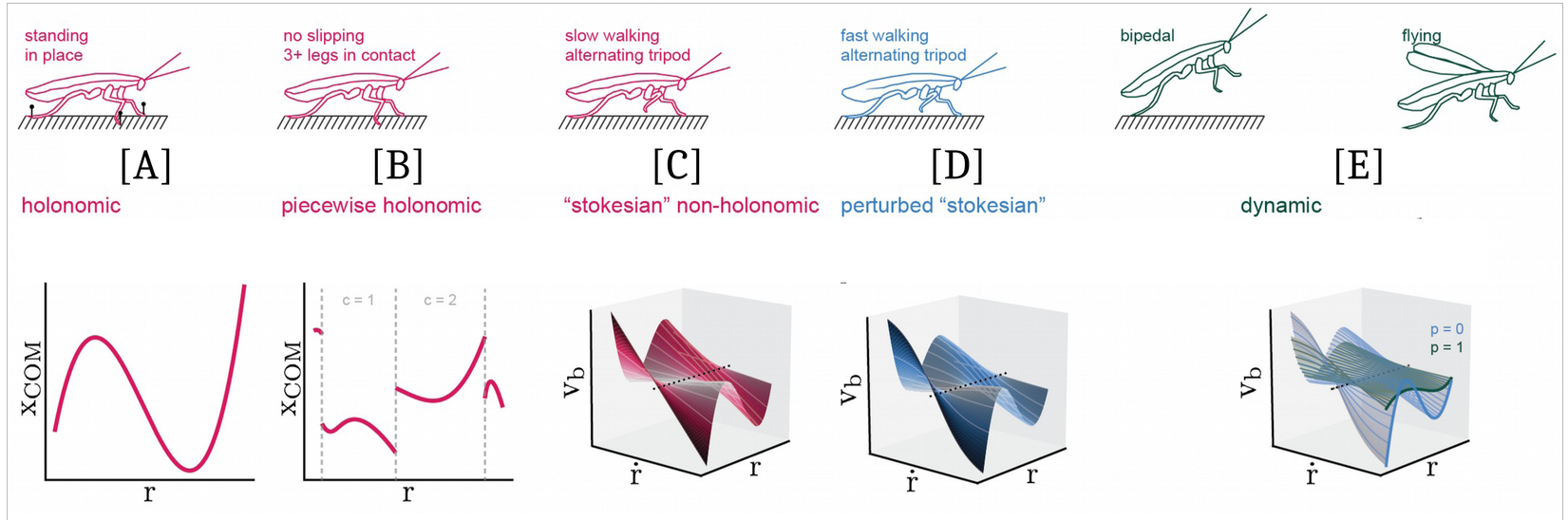

Fig. 1

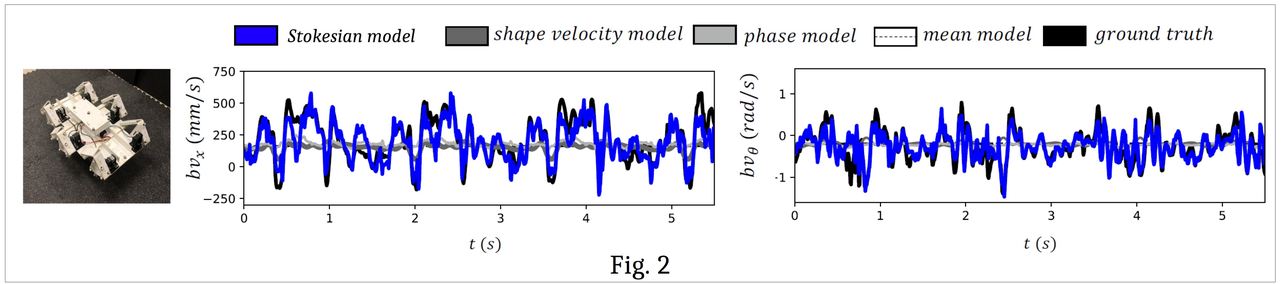

Fig. 2

Conclusion

The theory of geometric mechanics [?] tells us that **ALL** locomotion follows the "reconstruction equations" (Fig. 2 [E]) but in practice for almost all multi-legged systems the Stokesian model is quite accurate.

This is mathematically provable for non-slip, piecewise holonomic motion. It is empirically observed even when slipping with Coulomb type friction.

Stokesian models are a unifying framework applying to all locomotion where substantial contact forces exist all the time – swimming, slithering, and walking.

References

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