Many Multi-Contact Collisions are Simpler Than Expected

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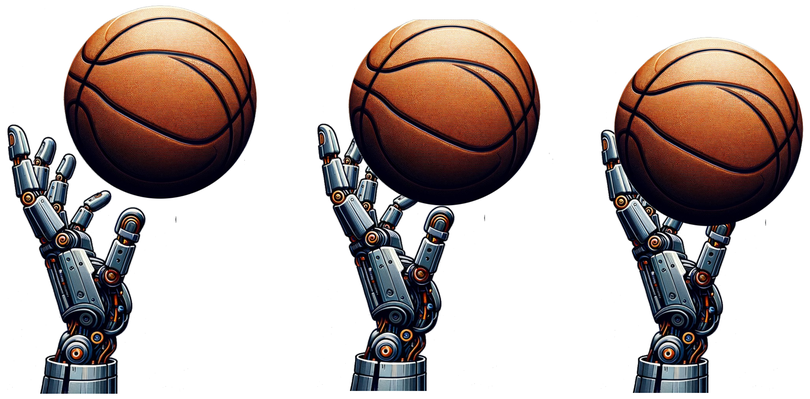
Problem Statement

Consider catching a frictionless basketball with 5 fingers (Fig. 1). We would expect the motion to be governed by $32=2^5$ distinct differential equations for each of the 32 contact states. Recent work on such "Event-Selected Systems" (ESS) [1,2] suggests that despite this, the governing equation is C1 smooth.



(a) (b) (c)

Fig. 1: One of the 5!=120 flight (a) to stance (c) transitions that can occur when catching a basketball one-handed.

When is a System "Event-Selected"?

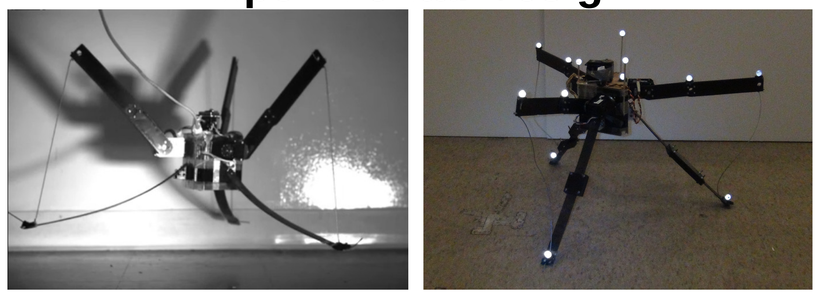
"Event-Selected" is a **local** property of the flow for some hybrid dynamical systems [1]. An open ball in a state space is event-selected with M events when

- The ball is partitioned into 2^M pieces by M guards
- The vector field in each piece is smooth
- The guards all intersect at a point
- Each guard is a proper zero level set
- Each event occurs once and does not repeat

Sufficient Condition for Smoothness

When the dynamics are **event-selected** and **second-order** such that the state vector can be split into "positions" and their "velocities," and the **guards depend exclusively on the positions**.

Experimental Design



(a) (b)

Fig. 2: Our three-legged hopper executing an open-loop event-selected hopping gait (a) and decorated with motion capture markers (b).

Experimental Results

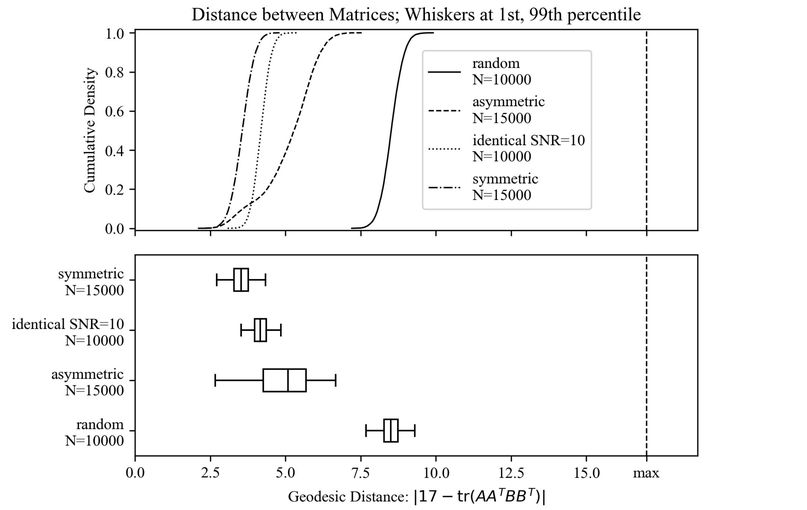


Fig. 3: Distances between bootstrapped impact map matrices in our symmetric and asymmetric hopper motion datasets versus random and noised identity matrices of the same shape. The self-similarity of the impact map matrices suggests that the hopper impact dynamics are C1 smooth.

Motion Capture Processing Pipeline

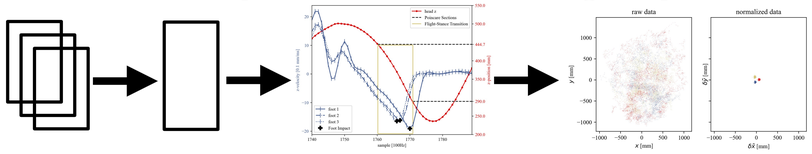


Fig. 4: Data processing pipeline steps. From left to right: trial processing, aggregation, contact sequencing, and coordinate normalization.

Leveraging ESS for Rapid Integration

Motivated by our experimental results, we were able to leverage the results of [2] to develop a general-purpose numerical integrator for multi-contact systems [3].

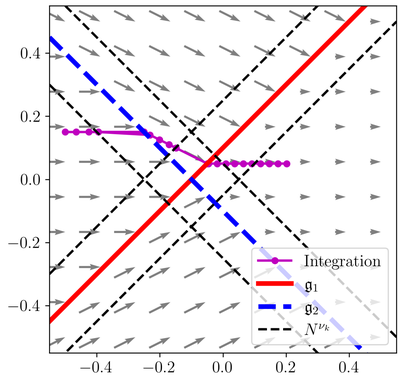


Fig. 5: Illustration of the integration "projection step" on a toy ESS.

Fig. 6: Link to [3] source code

Future Work

We are developing models to evaluate the performance of ESS in model-predictive control and differentiable programming settings.

References

[1] "Event-selected vector field discontinuities yield differentiable flows". S. A. Burden, et al., SIAM Journal of Applied Dynamical Systems, 2016.

[2] "Representing and Computing the B-Derivative of the Piecewise-Differentiable Flow of a Class of Nonsmooth Vector Fields". G. Council et al., Journal of Computational and Nonlinear Dynamics, 2022.

[3] "Rapid Integrator for a Class of Multi-Contact Systems". M. Anderson et al., arXiv preprint, 2024.

Acknowledgments

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